Comparison of probabilistic and deterministic optimizations using genetic algorithms

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Abstract This paper describes an application of genetic algorithms to deterministic and probabilistic (reliability-based) optimization of damping augmentation for a truss structure. The probabilistic formulation minimizes the probability of exceeding upper limits on the magnitude of the dynamic response of the structure due to uncertainties in the properties of the damping devices. The corresponding deterministic formulation maximizes a safety margin with respect to these limits. Because this work was done in the context of an experimental comparison of the reliabilities of the resulting designs, antioptimization was used to maximize the contrast between the probabilities of failure of the two designs. This contrast maximization was also performed with a genetic algorithm. The paper describes the genetic algorithm used for the optimization and antioptimization, and a number of modifications to the antioptimization formulation intended to reduce the computational expense to an acceptable level. Optimal designs are obtained for both formulations. The probabilistic design shows a very significant improvement in reliability.

1 Introduction

Structural design optimization is usually based on analytical models. Performance levels predicted by these models correspond to nominal values of geometric, loading and material properties. However, these properties are random in nature and generate scatter in the performance of the actual system.

Design optimization problems typically involve a large number of performance requirements. Deterministic optimization tends to make several requirements critical in a deterministic sense (i.e. for the nominal system). Since the property variations are neglected, this may increase the number of possible failure modes, which in turn tends to increase the probability for the actual system to violate one or more of these requirements. Large safety factors are used to reduce this probability. Unfortunately, these safety factors are not systematically adjusted to discriminate between requirements with different magnitudes of scatter or different costs. As a result, deterministic optimization can lead to either unreliable or overdesigned systems (see e.g. Moses and Kinser 1967; Moses 1977).

Probabilistic optimization, on the other hand, attempts to use some knowledge about the statistics of the uncertainties in the system to predict – and minimize – the probability of violating the design requirements. This approach is gaining popularity and has been shown in numerical simulations to provide safer designs (e.g. Parimi and Cohn 1978; Yang et al. 1990; Sepulveda 1994).

In this paper, we compare optimal designs obtained from deterministic and probabilistic formulations of the same design problem. Both optimizations are performed using genetic algorithms (GA). This comparison is performed as part of a program for experimental validation of probabilistic optimization that was the focus of a previous publication by Ponslet et al. (1994) and Ponslet (1994). In this context, it is important to have a large difference between the probabilities of failure of the two designs (because a small difference would be difficult to measure). To achieve this, we formulate a contrast maximization problem that finds a problem that maximizes the difference in probabilities of failure between the two designs. Some parameters of the deterministic and probabilistic optimization problems are used as the de-
sign variables in this contrast maximization. This process can be seen as antioptimization (e.g. Haftka and Kao 1990; van Wamelen et al. 1993), in the sense that it makes the deterministic design look as bad as possible compared to the reliability-based design. The antioptimization is also solved using a genetic algorithm.

The design case used throughout this paper is that of a truss structure with damping augmentation where the main source of response uncertainty is the variability in the properties of the nominally identical damping devices. The uncertainties are quantified by measuring the properties of a large number of dampers.

In the rest of the paper, we first describe the truss and dampers used in this study. We then describe the design problem and the models and analysis techniques for estimating the vibration amplitude and probability of failure of a truss design. The alternative formulations of the deterministic and probabilistic optimization problem are presented next. We then describe in detail the genetic algorithm used in this study, followed by the formulation of the antioptimization problem. Finally, optimization results are presented and the deterministic and probabilistic optimal designs are compared.

2 System description

The structure is shown in Fig. 1. It is a short, beam-like truss assembled from aluminium members and nodes. The truss is about 1 meter long and weighs about 4.4 kg. The same truss was the subject of a previous experimental study of the scatter in natural frequencies due to uncertainties in the masses and stiffness of the members and in the masses of the nodes (Ponslet et al. 1993).

![Laboratory truss](image)

Fig. 1. Laboratory truss

Figure 2 shows a plot of the magnitude of an experimental frequency response function (FRF) from excitation force to response acceleration for that truss. The first three modes are well separated. Their natural frequencies are about 100, 130 and 193 Hz. We also observe that the inherent damping in the second mode is much higher than in the other two modes. For that reason, only modes 1 and 3 are considered in this work.

This structure is equipped with tuned vibration absorbers to reduce the dynamic response of the first and third modes. The tuned dampers (Fig. 3) consist of symmetric cantilevered beams, attached by their middle point to a node of the structure and carrying adjustable tip masses. Their first bending mode is tuned to a natural frequency of the truss to obtain a tuned damper behaviour. Two slightly different versions of this damper design (we will refer to them as type 1 and type 3) are used to target the first and the third natural frequency of the structure. The reader is referred to the work of Ponslet et al. (1994) and Ponslet (1994) for a detailed description of these dampers.

Although the mass of the dampers is very small compared to the mass of the structure (about 10 grams, or 0.2% of the total mass of the truss), they provide significant damping to the truss. Figure 4 shows the frequency response function of a tip node acceleration, before and after adding one type-3 damper to the structure (the damper was tuned to the frequency of mode 3 and located at the node and in the direction that correspond to the largest amplitude of vibration in the third mode shape). The reduction in amplitude achieved in mode 3 is more than 25 dB.

Note that the tuned damper does not significantly affect the other modes of the truss because the natural frequencies of these other modes are far from the tuned frequency of the damper, preventing any significant exchange of energy between the truss and the damper. This implies that at least...
one damper per target mode is needed. Also, the natural frequency of the damper is the most important parameter that determines its effectiveness.

3 Design problem

The design requirements are upper limits on the accelerations, at given points on the structure within prescribed frequency ranges for a given excitation. The locations and directions of the excitation and the response measurements are shown in Fig. 5. The displacement in modes 1 and 3 was maximum in these locations and directions. The excitation is assumed to have a flat spectrum and a unit amplitude. This allows us to use transfer functions from excitation force to response accelerations as a substitute for the accelerations themselves. The transfer functions $H^{(1)}$ and $H^{(3)}$ corresponding to the two measurement points are defined as

$$H^{(1)}(i\omega) = \frac{\ddot{u}^{(1)}(i\omega)}{f(i\omega)},$$

$$H^{(3)}(i\omega) = \frac{\ddot{u}^{(3)}(i\omega)}{f(i\omega)},$$

where $u^{(1)}$ and $u^{(3)}$ are displacements, $f$ is the excitation force, $\omega$ is the frequency, $i = \sqrt{-1}$, and a dot denotes differentiation with respect to time. Superscripts of transfer functions and displacements denote the corresponding vibration mode.

The upper limits on $H^{(1)}$ and $H^{(3)}$ are set to the same level $H_{\text{lim}}$ [i.e., $H^{(1)}_{\text{lim}} = H^{(3)}_{\text{lim}} = H_{\text{lim}}$] in two frequency windows, covering the two modes of interest (mode 1 and mode 3). This is illustrated in Fig. 6.

We consider the following design problem. The truss has been designed with one damper for each mode (modes 1 and 3 only) to limit dynamic response accelerations. A large number of trusses and dampers have been manufactured and samples have been tested. The tests have revealed a significant mistuning of the dampers that will result in poor overall performance of the damped structure. We assume that the dampers cannot be modified to improve their tuning. However, tuning masses can be easily added to the nodes of the truss to modify its natural frequencies and improve tuning. These tuning masses have a fixed magnitude (16.6 g) and in order to limit the added weight—a maximum of 10 masses can be used. Also, the locations of the two dampers can be modified. The problem consists of optimally redesigning the system by adding tuning masses and moving the dampers to ensure satisfactory performance.

The properties of the truss are assumed deterministic. The parametric uncertainties in the system are limited to the properties of the tuned absorbers and the different realizations of the system are created by attaching different dampers to the same truss.

4 Models and analysis techniques

A complete description of the analysis techniques and approximations can be found in Ponslet’s dissertation (Ponslet 1994). Here we give a brief overview.

4.1 Truss model

The truss is modeled using 3-dimensional rod finite elements. Complex stiffnesses are used to model inherent damping. Support springs are included to account for the flexibility of the wall support. All values of the masses of the members and nodes and of the stiffnesses of the members are equal to the mean values of series of measurements performed on a large number of members and nodes. Details on those measurements can be found in the paper by Ponslet et al. (1993). The model contains 36 degrees of freedom.
4.2 Tuned damper model
A two-degree-of-freedom (d.o.f.) model of the dampers was devised. The simplified model consists of a spring-mass system with complex stiffness. It is completely defined by four parameters: the total mass \( m_T \), the tip mass \( m \), the natural frequency \( \omega_n \), and a loss factor \( \eta \). A least-square parameter identification technique is used to measure those parameters (see Pouslet et al. 1994).

To include the tuned damper in the finite element model of the truss, we created a special 4 d.o.f. element (3 are shared with the existing model of the truss). With two dampers on the truss, the number of degrees of freedom is increased to 38.

4.3 Approximation techniques
The evaluation of the transfer functions requires the first few eigenvalues and eigenvectors of the truss with tuning masses and dampers. Obtaining exact eigenvectors and eigenvalues for a truss with two dampers requires solving a generalized complex eigenproblem with 38 degrees of freedom. The computational cost associated with this operation is considered prohibitive for our purposes.

To reduce computational expenses, we implemented an approximate analysis technique. The method consists of modelling each damped mode with a single mode shape of the truss connected to the model of the damper. An analytical expression for the frequency response function excitation force to response acceleration is easily derived for the resulting 2 d.o.f. model. This analysis is repeated with a different basis for each mode of interest. It is about 50 times faster than a full analysis and the accuracy is sufficient for our design purposes.

4.4 Probabilistic analysis
The complex, nonlinear relationship between the parameter of a tuned damper and the magnitude of the peak response of a truss equipped with that damper and the existence of multiple most probable failure points prohibits the use of second moment methods to estimate failure probabilities. For that reason, we use Monte Carlo simulations to estimate the probabilities of failure. A sample size of 1000 was chosen. Because the total mass of the dampers has very little scatter, it is assumed deterministic in the simulations. The three remaining parameters (natural frequency \( f_n \), tip mass \( m \) and loss factor \( \eta \)) of each damper are assumed normally distributed, correlated random variables. A complete 1000 point Monte Carlo simulation uses about 10 seconds of CPU time on an IBM 3090 computer in vectorized mode.

5 Deterministic and probabilistic design formulations
The optimization problem consists of finding optimal locations for up to 10 tuning masses and optimal locations and directions for 1 damper of each type in order to satisfy the limits on the magnitude of the transfer function at the specified response measurement points. The deterministic and probabilistic optimizations use the same design variables: the locations of the two dampers \( (a_1, a_3) \) for the type 1 and type 3 damper, respectively, the orientations of these two dampers on the node \( (d_1, d_3) \), and the locations of a maximum of 10 masses \( (m_1, \ldots, m_{10}) \), for a total of fourteen design variables.

The locations of the masses and dampers are restricted to the nodes of the truss. Moreover, to limit the size of the design space and reduce the CPU time needed for the optimization, we restrict the candidate locations for the masses and dampers to subsets of the twelve nodes of the truss. The dampers are only allowed to be attached to nodes 10, 11 and 12 at the tip of the beam-truss, because a preliminary study showed that they were more effective when attached to those nodes. The masses can be attached to nodes 0 and 7-12, because we found that they were not effective when placed at the remaining nodes. Node 0 is fictitious (masses attached to that node have no effect on the dynamic response) and is used to allow for less than 10 masses on the truss. Since a total of up to ten masses can be attached to six locations, any node can support more than one mass at a time. We use the standard mounting holes of the nodes to attach the dampers. Those holes are aligned in nine discrete directions, numbered 1 to 9. Note that all fourteen design variables are discrete. Discrete variables are not handled easily by conventional gradient-based optimization methods, so we use a genetic algorithm as the optimizer.

With these variables, the deterministic optimization is formulated as

\[
\begin{align*}
\text{maximize} & \quad \alpha = H_{\text{lim}} - \max \left[ H_{\text{peak}}^{(1)}, H_{\text{peak}}^{(3)} \right], \\
\text{subject to} & \quad m_1, \ldots, m_{10} \in \{0,7,8,9,10,11,12\}, \\
& \quad a_1, a_3 \in \{10,11,12\}, \\
& \quad d_1, d_3 \in \{1,2,3,4,5,6,7,8,9\}, \end{align*}
\]

with \( a_1, a_3 \in \{10,11,12\}, \quad d_1, d_3 \in \{1,2,3,4,5,6,7,8,9\}, \quad m_1, \ldots, m_{10} \in \{0,7,8,9,10,11,12\}, \)

where \( H_{\text{peak}}^{(1)} \) and \( H_{\text{peak}}^{(3)} \) are evaluated for nominal (mean) damper properties. Recall also that \( m_i \) denote the locations of the masses rather than their magnitudes. A negative value of the safety margin \( \alpha \) corresponds to a design that violates at least one of the constraints on response accelerations. Note that the use of the max function leads to discontinuities in the slope of the objective function which would cause numerical difficulties with gradient-based optimizers. However, such discontinuities are not a problem when genetic algorithms are used for the optimization.

The corresponding probabilistic formulation minimizes the probability of failure \( P_f \), which is the probability that the design will violate any of the two requirements due to scatter in the damper properties. It is given as

\[
\begin{align*}
\text{minimize} & \quad P_f = P \left[ H_{\text{peak}}^{(1)} \geq H_{\text{lim}} \text{ or } H_{\text{peak}}^{(3)} \geq H_{\text{lim}} \right], \\
\text{subject to} & \quad a_1, a_3 \in \{10,11,12\}, \\
& \quad d_1, d_3 \in \{1,2,3,4,5,6,7,8,9\}, \\
& \quad m_1, \ldots, m_{10} \in \{0,7,8,9,10,11,12\}. \end{align*}
\]

The genetic algorithms used to solve problems (3) and (4) are described in the next section.
6 Genetic algorithm

The same genetic algorithm is used for the solution of the deterministic and probabilistic formulations; only the objective function differs. We use a string of fourteen integer numbers as the chromosome of the genetic search. Each gene in the chromosome stores the value of one of the fourteen design variables. Symbolically, the string can be written as:
\[ \text{chromosome} = [a_1d_1a_3d_3m_1 \ldots m_{10}] \tag{5} \]

using the previously defined notations. The ranges of the variables are as shown in (3) and (4). The size of the design space is approximately equal to \(2 \times 10^{11}\).

In the implementation of the algorithm, the gene values are all integer numbers varying from 1 to the number of possible physical values of the corresponding design variable. For the damper locations, for example, genes \(a_1\) and \(a_3\) in the chromosome actually take the values 1, 2 or 3 instead of 10, 11 and 12. This simplifies the implementation of the genetic operators. A correspondence table is used to convert gene values into values of the design variables.

The algorithm uses the three classical genetic operators (selection, crossover, and mutation) and an elitist strategy. The algorithm applies these operators to a population of \(n\) designs (with \(n\) typically equal to 20 in this work). A random initial population is used, and the search is stopped after a fixed number of generations.

**Elitist strategy.** The elitist strategy consists of always cloning the best individual of the current population into the next population. This guarantees that the best design found will never be lost in later stages of evolution.

**Selection.** Parent designs are selected randomly using the ranking technique. The designs are first evaluated and ranked in decreasing order of objective function. The probability of selection of a particular design is then proportional to the population size minus its rank in the population.

**Crossover.** Parent designs are mated using a one-point averaging crossover. A continuous, real random number is used as the cutting point; the parts of the strings on either side of the cutting point are exchanged between parents and the remaining gene is generated as a weighted average of the two parent genes. For example, assume the following two parent chromosomes have been selected for reproduction:

**parent 1:** 27197751783143,

**parent 2:** 12153114227145.

Further assume that we have picked a random cutting point equal to 4.31. An offspring chromosome is formed using genes 1 to 4 from parent 1, and genes 6 to 14 from parent 2. The value of gene number 5 is computed as weighted average,

\[ 0.31 \times 7 + 0.69 \times 3 = 4.24 \tag{6} \]

and rounded to 4. Note that this operation is capable of introducing gene values that were not present in any of the parents. Creating new genetic information is vital to avoid creating “blind spots” in the design space. The resulting offspring chromosome is then:

**offspring:** 27194114227145.

The offspring has some of the characteristics of each parent design and also some new features (gene 5). The crossover is performed with a given probability \(P_c\). If crossover is not used, one of the two parent chromosomes is chosen randomly and cloned into the new population.

**Mutation.** The mutation is a classical gene by gene random mutation: for each gene in a child design, a probability of mutation \(P_m\) is applied. If mutation occurs, a random value with uniform distribution within the range of the particular gene is substituted to the current value of that gene.

Besides the choice of a particular coding, the values of the probability of mutation \(P_m\) and the population size \(n\) are important parameters that affect the convergence and reliability of the search. Unfortunately, there are no general rules defining appropriate values for these parameters. In fact, their optimum values are so dependent on the particular optimization problem that they must be determined by trial and error. This is an expensive proposition, since it requires a large number of runs to determine the average performance as a function of the parameters.

We roughly tuned these parameters by running the algorithm ten times for each combination of parameters. The values that maximized the reliability of the search were retained. The total number of analyses per search was set to 1200 to limit the computational expense. The population size, and mutation and crossover probabilities were set to \(n = 20\) individuals, \(P_m = 6\%\), and \(P_c = 95\%\), respectively. The probability that an individual design will undergo at least one mutation is \(1 - (1 - 0.06)^{14} = 0.58\).

Because genetic algorithms (GA) are random search techniques, a single search does not consistently provide an optimum design. Instead, it provides a design that has a certain probability of failing within near the optimum. Clearly, the reliability can be considerably improved by running the genetic search a few times on any given problem (with different seeds used in the random number generators). The best design of those multiple searches has a higher probability of being very close to optimum: if \(r\) is the reliability of a single search (i.e. the probability that the final design is within a specified margin from the actual optimum), then the reliability of \(m\) searches is

\[ r_m = 1 - (1 - r)^m. \tag{7} \]

This shows that the reliability of the genetic optimization grows very fast as the number of searches goes up. We evaluated the reliability of a single search to be about 30%, by performing a number of identical genetic optimizations and counting the times that the optimal design obtained was close to the best known design (designs were considered close if the value of their objective function was within 1% of the best known design). Since the deterministic optimization is cheap, we run it ten times and achieve a reliability of about 97%. The probabilistic optimization, on the other hand, includes Monte Carlo simulations and is therefore much more expensive. We cannot afford to run it ten times. However, with three runs, the reliability is already about 66%.

7 Contrast maximization

The goal of our project is to evaluate experimentally the difference in reliability between deterministic and probabilistic designs. Measuring probabilities of failure in the laboratory is timeconsuming, and measuring very small probabilities or
very small differences in probabilities of failure requires a prohibitive large number of experiments. For this reason, we had to identify a design problem that leads to alternative designs with a large difference in probability of failure. To find such a design problem we carried on a process known as antioptimization or contrast maximization, which maximizes the contrast between the two design approaches.

Antioptimization requires a number of problem parameters that can be varied to achieve the increased contrast. For our problem, we selected the following parameters: the mean values of the natural frequencies of the two types of dampers (the amount of mistuning), the failure limit $H_{\text{lim}}$ (the stringency of the requirements), and the relative magnitudes of the scatter in the parameters of the two types of dampers.

In a problem with two (or more) failure modes, a probabilistic approach can provide designs that are significantly different from deterministic designs by taking advantage of cost and/or scatter differences between the two failure modes. If the costs of controlling the two failure modes are the same, but the magnitudes of scatter are different, then the probabilistic optimization will provide a larger safety margin to the large-scatter failure mode. If the magnitudes of scatter are similar, but the costs of controlling the different failure modes are different, then the probabilistic optimization will provide a larger safety margin to the cheaper mode. These effects accumulate if the design problem is such that the cheapest mode also has the largest scatter.

Because added masses have a greater effect on high frequency modes than low frequency modes, it is easier to control the natural frequency of the third mode of our truss with tuned masses than the first mode (so that the third mode is “cheaper” to control). It is therefore desirable to have more scatter in the properties of type-3 dampers than in those of type-1 dampers. Based on preliminary measurements on prototypes of the dampers and on hardware limitations, we selected coefficients of variation of 1.5% and 3.5% in the natural frequencies of the type-1 and type-3 dampers, respectively. The standard deviations of all damper parameters were then kept constant in the antioptimization for the failure limit and damper frequencies.

To find an appropriate combination of the other problem parameters (mean natural frequencies of the two types of dampers and acceleration limit), we incorporated these variables into a contrast maximization formulation. Since both the deterministic and the probabilistic designs are solutions of discrete optimization problems, the optimum probabilities of failure of those designs are discontinuous functions of the problem variables. For that reason, the antioptimization was itself performed using a genetic algorithm. Since our genetic algorithm uses integer coding, we first discretized the three problem parameters. The mean natural frequencies $f_n^{(1)}$ and $f_n^{(3)}$ were allowed to vary between 90 and 110 Hz in steps of 2 Hz (11 values) and between 182 and 202 Hz in steps of 2 Hz (11 values), respectively (this corresponds to a ±10 Hz mistuning range). The acceleration limit $H_{\text{lim}}$ was varied between 2 and 9 m/s$^2$N in steps of 0.5 m/s$^2$N. Using antioptimization, we can identify values for these three problem parameters that maximize the difference in probability of failure between a deterministic optimum design and a probabilistic optimum design. The antioptimization problem is formulated as

$$\max \Delta P_f = P_{f\text{det}}^* - P_{f\text{prob}}^*,$$

where

$$f_n^{(1)}, f_n^{(3)}, H_{\text{lim}}$$

where $P_{f\text{det}}^*$ and $P_{f\text{prob}}^*$ are the probabilities of failure of the deterministic and probabilistic optima corresponding to formulations (3) and (4), respectively. Conceptually, formulation (8) implies embedding complete deterministic and probabilistic optimization inside a higher level optimization. The flow chart of the antioptimization is shown in Fig. 7.

![Flow chart of antioptimization](image)

The outside loop corresponds to the antioptimization and searches for the optimal values of the three problem parameters $f_n^{(1)}, f_n^{(3)}$, and $H_{\text{lim}}$. The probabilistic and deterministic optimizations are embedded within that loop and provide probabilistic and deterministic optimal designs corresponding to the current values of the problem parameters. From those two optimal designs, the objective function of the outer loop is evaluated as the difference between the probabilities of failure evaluated with a Monte Carlo simulation (note that the Monte Carlo simulation is done within the inner optimization loop in the case of the probabilistic design).

Since genetic algorithms are used as optimizers and the probabilities of failure are evaluated with Monte Carlo simulations, we cannot afford such a two-level optimization. With 1000 analyses per Monte Carlo simulation and about 1200 design evaluations for each design optimization, assuming that about 1200 iterations will be needed to reach convergence at the antioptimization level, the flow chart of Fig. 7 represents more than a billion analyses! To avoid this, we would want
to combine the three problem parameters of (8) with the
design variables of the probabilistic optimization (4) so that the
inner probabilistic optimization loop is eliminated.

As a first step towards this goal we take advantage of
the relatively low cost of the deterministic optimization (low
because the Monte Carlo simulation is not in the loop). Addi-
tionally, the deterministic optimum design is independent of
the failure limit. This is because the same acceleration limit
\( H_{\text{lim}} \) is used for both modes so that maximizing the safety
margin \( \alpha \) is equivalent to minimizing the largest of the two
peak accelerations. With this, the deterministic optimum de-
dpends only on the natural frequencies \( f_{n}^{(1)} \) and \( f_{n}^{(3)} \). Since
these frequencies can each take eleven different values, there
are only \( 11 \times 11 = 121 \) distinct deterministic design problems
in the antioptimization space. We solved all 121 determinis-
tic optimization problems in advance and stored the results
in a database for use by the antioptimization.

As noted before, the result of genetic search is partly ran-
dom; there is no guarantee of optimality. Occasionally, a
single search can lead to a design that is far from optimal.
This is not acceptable in the antioptimization: if for one par-
ticular combination of problem parameters the deterministic
design in the database is far from optimal, its probability
of failure will be abnormally high. This will induce the an-
tioptimization to falsely identify that problem as one that
maximizes the contrast. This is illustrated in Fig. 8 where
we plotted the final value of the peak acceleration for 11 of
the 121 deterministic design problems.

![Fig. 8. Peak acceleration of optimum deterministic designs for
11 values of the natural frequency of the type-1 damper (type-
3 damper tuned at 182 Hz); for each problem the result from a
single search is compared for robustness to the best of ten genetic
searches.](image)

The eleven problems correspond to the eleven candidate
values for the natural frequency of the type-1 damper (the
natural frequency of the type-3 damper was set to 182 Hz in
all eleven problems). The solid line with black dots corre-
sponds to designs obtained with a single run of the GA for
each problem. The designs plotted on the other four lines are
obtained as the best result out of 10 GA runs, for each prob-
lem. This process was repeated four times to evaluate the
robustness of the final designs. We observe that, although
one search occasionally finds very good designs (for 90, 96
and 102 Hz, for example), it also finds designs that have as
much as 20% more peak acceleration than the optimum (for
example at 94 and 98 Hz). Using the "best of 10 runs" strat-
egy on the other hand, the designs obtained are consistently
within about 2% of the optimum.

![Fig. 9. Peak acceleration of optimum deterministic designs for
121 problems; for each problem the "optimum" was obtained as
the best of 10 genetic searches.](image)

All 121 problems were optimized using this strategy and
the resulting designs were stored in a database. The three-
dimensional graph of Fig. 9 represents the peak amplitude of
the 121 deterministic optima as a function of the two
problem parameters \( f_{n}^{(1)} \) and \( f_{n}^{(3)} \). Monte Carlo simula-
tions with 1000 replications were performed on those 121 op-
timum deterministic designs and the probabilities of failure
were evaluated for all candidate values of the failure limit
\( \{2, 2.5, 3, 3.5, \ldots, 9\} \) and also stored in the database. This
database represents a function \( P_{f}^{\text{det*}}[f_{n}^{(1)}, f_{n}^{(3)}, H_{\text{lim}}] \) which
provides the probability of failure of the optimum determin-
istic design for each combination of values of the antiopti-
mination variables.

Given this function, we can now combine the antiopti-
mination and probabilistic optimization into a single opti-
mization loop:

\[
\text{maximize } \Delta P_{f} = P_{f}^{\text{det*}} - P_{f},
\]

\[
f_{n}^{(1)}, f_{n}^{(3)}, H_{\text{lim}},
\]

\[
m_{1}, \ldots, m_{10}.
\]
The resulting optimization loop in (9) then *simultaneously* looks for a problem and the corresponding probabilistic design that maximize the difference in probability of failure compared to the optimum deterministic design corresponding to the same problem, where $P_f$ is now the probability of failure of the current problem/probabilistic design. Specifically, the optimizer in (9) simultaneously finds (i) the values of $a_1, d_1, a_3, d_3, m_1, \ldots, m_{10}$ that minimize $P_f$ for given values of $f^{(1)}_n, f^{(3)}_n, H_{lim}$ and (ii) the values of $f^{(1)}_n, f^{(3)}_n, H_{lim}$ that maximize the difference of $P_{det*}$ and $P_f$. Compared to the conceptual formulation of Fig. 7, the number of analyses is reduced by three orders of magnitude.

With this formulation, the antioptimization is reduced to a single loop that combines the contrast maximization and the probabilistic optimization. The deterministic optimization has been eliminated and replaced by a simple table lookup. Figure 10 shows the corresponding flow chart. The only significant computational effort left inside the loop consists of a 1000 point Monte Carlo simulation to evaluate the probability of failure of the current design.

**Fig. 10. Antioptimization, final flow chart**

The design variables and ranges are left unchanged. With this new formulation, the probability of failure $P_f$ of the current problem/design is minimized $(0.5 - P_f)$, which is larger than 50%. This forces the failure limit to stay at reasonable levels, giving the algorithm a chance to improve the current design before trying to maximize the contrast with the deterministic design. The transition from one objective to the other is gradual, since it depends on the performance of each particular problem/design in the population: in the first few generations, most designs are evaluated using $0.5 - P_f$, while in later generations, the probability contrast is used for most designs.

The contrast maximization procedure identified the following problem as an ideal example: the dampers of type 1 would have an average natural frequency of 110 Hz (about 10 Hz overtuned), type-3 would have a mean natural frequency of about 182 Hz (about 11 Hz undertuned), and the failure limit $H_{lim}$ would be equal to 6.5 m/s^2 N. For this problem, the deterministic design has a probability of failure of 18.6% compared to only 2.5% for the probabilistic design. This corresponds to a difference of 16.1% between the probabilities of failure. The design problem corresponds to a situation where one type of damper is overtuned and the other undertuned. The optimum deterministic and probabilistic designs represent different compromises between improving the behaviour of one mode and degrading the other (since adding masses can only reduce the natural frequencies of the truss).
8 Optimization results

In the contrast maximization, we have assumed that we could control the mean values and standard deviations of the parameters of the two types of dampers. For the experimental validation, we created samples of dampers that match the desired properties as closely as possible. The samples were then measured and experimental statistics of the damper parameters were obtained. They are listed in Tables 1 and 2. Details about the measurements can be found in a paper by Ponslet et al. (1994).

Table 1. Type 1 dampers, statistics of parameters (sample of 29)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$m$</th>
<th>$f_n$</th>
<th>$\eta$</th>
<th>$m_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>7.075 g</td>
<td>109.97 Hz</td>
<td>0.12604</td>
<td>10.811</td>
</tr>
<tr>
<td>St. dev.</td>
<td>0.0788 g</td>
<td>1.7487 Hz</td>
<td>0.006772</td>
<td>-</td>
</tr>
<tr>
<td>C.O.V.</td>
<td>1.11%</td>
<td>1.59%</td>
<td>5.37%</td>
<td>-</td>
</tr>
<tr>
<td>Correlation coefficients</td>
<td>1.000</td>
<td>0.693</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Type 3 dampers, statistics of parameters (sample of 29)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$m$</th>
<th>$f_n$</th>
<th>$\eta$</th>
<th>$m_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>7.340 g</td>
<td>180.16 Hz</td>
<td>0.15811</td>
<td>11.528</td>
</tr>
<tr>
<td>St. dev.</td>
<td>0.0944 g</td>
<td>5.5609 Hz</td>
<td>0.009987</td>
<td>-</td>
</tr>
<tr>
<td>C.O.V.</td>
<td>1.26%</td>
<td>3.09%</td>
<td>6.32%</td>
<td>-</td>
</tr>
<tr>
<td>Correlation coefficients</td>
<td>1.000</td>
<td>0.697</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

A deterministic optimum was obtained with nominal values for the damper parameters taken equal to the means shown in Tables 1 and 2. The optimization was repeated ten times. The convergence history of the best of these ten runs is shown in Fig. 11. The figure shows the peak amplitudes of the transfer function of modes 1 and 3 of the best design in the current population of the genetic search as a function of the generation number, and the total number of objective function evaluations.

Because we minimize the largest of these amplitudes and we do not penalize for the number of tuning masses used in the design, the amplitudes of the two modes tend to equalize in the optimum design. The final value of the safety margin (using semianalytical approximation and with $H_{\text{lim}} = 6.5$ m/Ns²) is 2.062 m/Ns².

The deterministic design uses seven masses, of which six are attached to node 7 and one to node 10. The damper locations are the same as in the deterministic design. The nominal peak amplitudes and probabilities of failure are listed in Table 3. Note the large difference in probabilities of failure in each failure mode. This is because although the deterministic optimization uses a uniform safety margin for both failure modes, the magnitudes of the scatter in the two modes are different.

The probabilistic optimum design used all ten tuning masses. One of those masses is attached to node 7 and nine to node 10. The damper locations are the same as in the deterministic design. The nominal peak amplitudes and probabilities of failure are listed in Table 4. The safety margin for the probabilistic design (using semianalytical approximation and with $H_{\text{lim}} = 6.5$ m/Ns²) is 1.500 m/Ns². Notice that the probabilistic design has a substantially reduced safety margin compared to the deterministic design.

Figure 13 shows a graphical comparison of those two alternative designs. The figure shows the histograms of the peak amplitudes of the two failure modes for each design. The dashed vertical line in each histogram represents the failure limit ($H_{\text{lim}} = 6.5$ m/Ns²). For the deterministic design, all failures occur in the third mode, due to the large scatter in the response of that mode. The first mode, on the other hand, has been given too wide a safety margin; the tail of the distribution is away from the failure limit.

The probabilistic design represents another compromise
for the same design problem: the safety margin has been tailored to the magnitude of the scatter in each mode. Compared to the deterministic design, the tuning masses have been moved to locations that are more effective on mode 3 and a few more masses have been used. This has considerably reduced the nominal peak amplitude of the third mode while slightly degrading the response of the first mode. Notice also that the scatter in mode 3 has been reduced.

The probabilistic design uses only three more tuning masses than the deterministic one, which represents a less than 0.8% increase from the total mass of the deterministic design. The predicted reliability of the system has increased from 84.7% to 99.4%. Probabilistic design was more effective than deterministic, because it accounted for the importance of the large scatter of mode 3 in trying to reduce the probability of failure, whereas deterministic optimization neglected scatter.

The two designs were tested with multiple realizations of the dampers. As described by Ponslet et al. (1994), the experiment confirmed the analytical predictions.

9 Concluding remarks
We have presented a comparison of probabilistic and deterministic design approaches for a damped structure. Alternative designs corresponding to the same design problem have been obtained from deterministic and probabilistic optimization and their reliabilities have been compared. Antioptimization has been used to enhance the chances of success of an experimental comparison, by maximizing the contrast between the reliabilities of the two designs. A combination of genetic algorithms, efficient approximate analysis, and Monte Carlo simulations has been used in the optimization. For the antioptimization – also performed by a genetic algorithm – a number of alterations in the formulation have reduced the computational expense to a level comparable to that of a single probabilistic optimization. Throughout the study, the chances of optimality of the designs obtained by genetic search have been increased through repeated runs of the algorithm. The final results have shown that, in some cases, probabilistic optimization can provide very large gains in reliability over deterministic optimization. The main reason is that probabilistic optimization is more effective than deterministic optimization in accounting for differences in the scatter of random variables and the resulting differences in the importance of failure modes.

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